

# Modal Analysis of Supersonic Flow Separation in Nozzles

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# Presentation Outline

- ▶ Introduction
- ▶ Separated Nozzle Flow & Sideloads
- ▶ Modal Analysis
- ▶ Selected Results
- ▶ Implicit Compressible Flow Solver
- ▶ Summary



# Project Aim

Predict **sideloads** during startup **transients** using  
detailed numerical tools

Investigate the applicability of applying modal  
**decomposition** methods to data from numerical  
simulations of **separated nozzle flows**

# Nozzle Side-loads



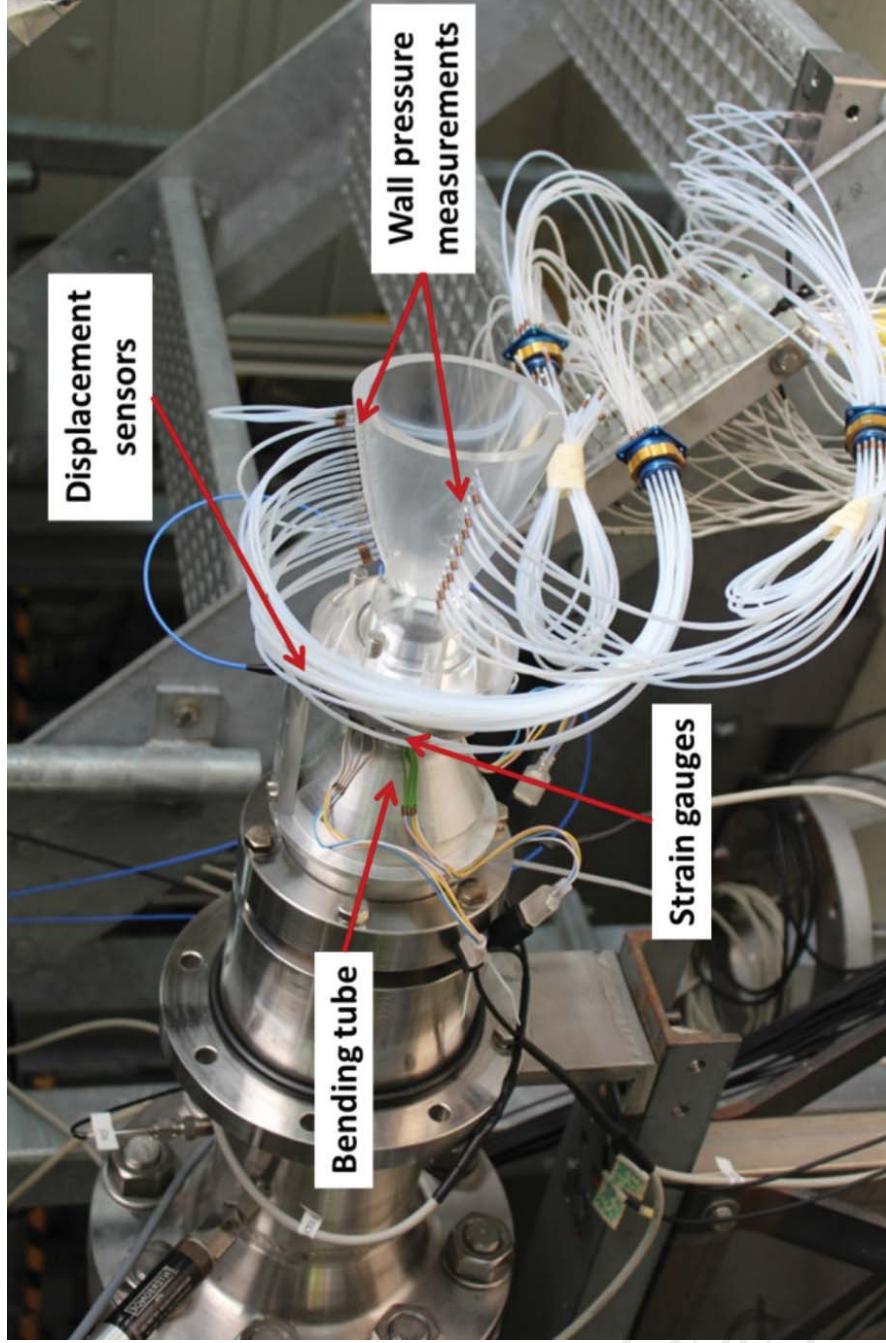
Space Shuttle Main Engine.

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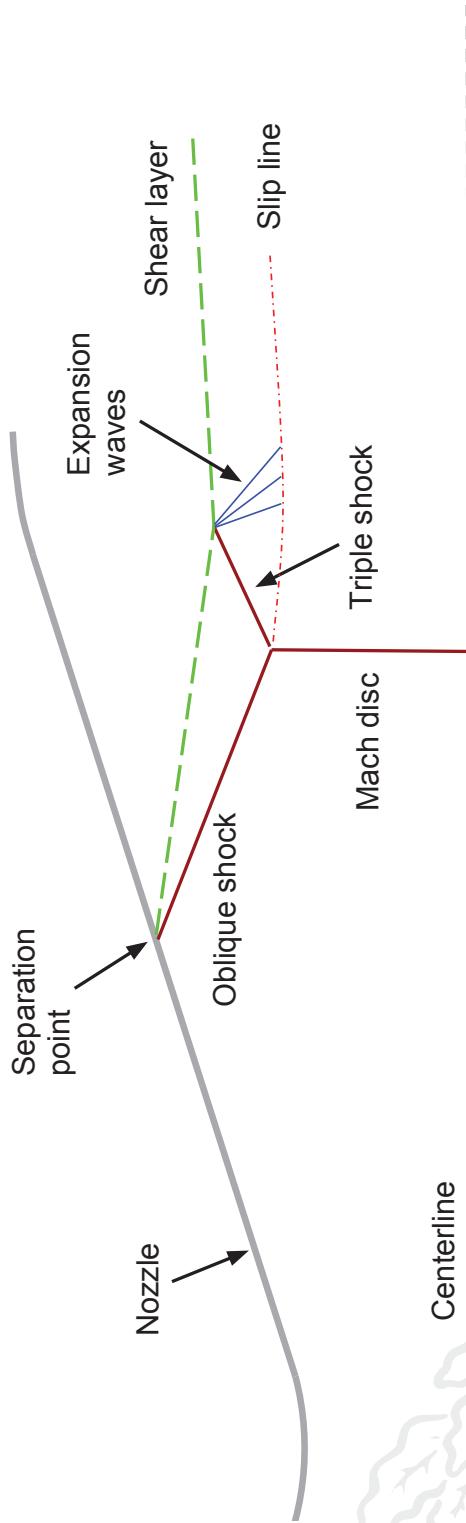


# Cold Supersonic Jet

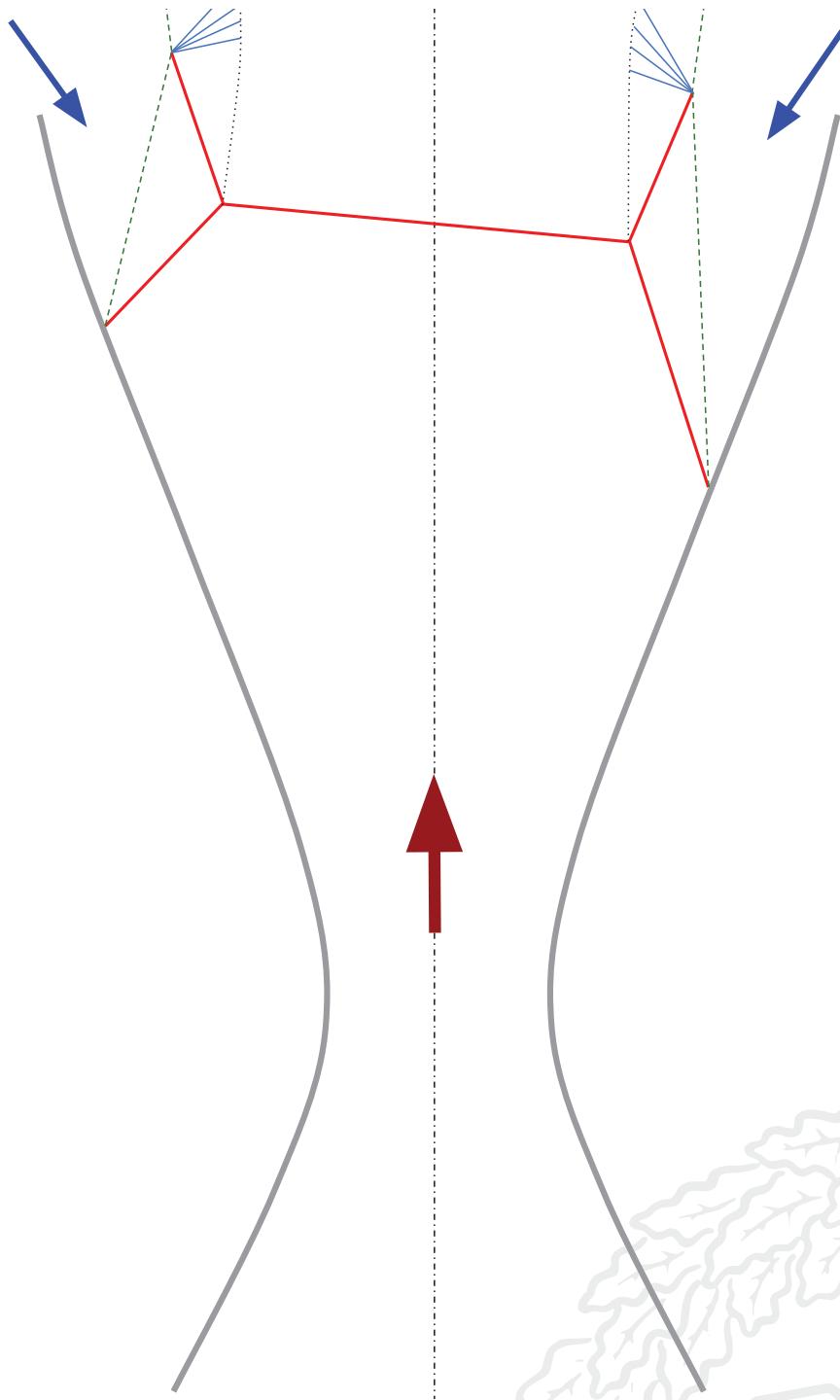


Experimental nozzle at DLR Lampholdshausen. From Génin and Stark 2016.

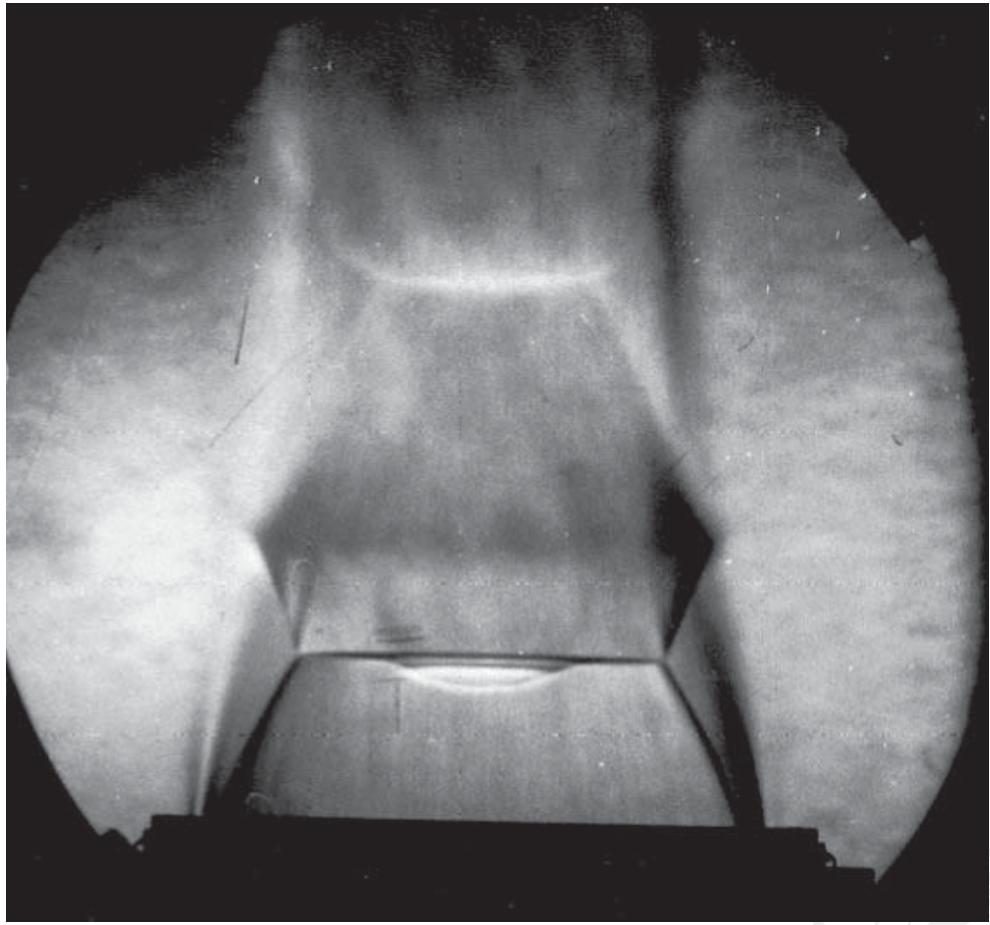
# Free Shock Separation



# Asymmetric Flow Separation



# Free Shock Separation - Schlieren Photograph



# Free Shock Separation - Numerical Simulation



Average Mach number in a Detached Eddy Simulation (DES).

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# Modal Analysis

Objective: To identify oscillating modes of a flow field.

Two different **modal decomposition** algorithms were used:

- Dynamic Mode Decomposition (**DMD**)
- **Arnoldi** algorithm



# Linear Flow Dynamics

Assume **linear mapping**,  $B$ , of the flow dynamics such that

$$BQ(t) = Q(t + \Delta t)$$

Where  $Q(t)$  is the flow field at time  $t$ .



# Orthogonal Subspace Projection

An **orthogonal vector basis** is obtained:

$$V = \{v_0, v_1, \dots, v_{m-1}\},$$

We seek a **projection of the flow dynamics** onto this basis:

$$V^T B V = C_m$$



This is done **without direct use of  $B$** , which is unknown in its matrix form.

# Eigendecomposition

We can then perform an **eigendecomposition** on the reduced-order matrix  $C_m$

The result is a set of modes, each with a fixed **frequency** and **decay/growth** rate and a coherent structure that can be visualized



# Arnoldi Algorithm

Uses a **linearized flow solver** to build a so called Krylov subspace projection basis



# Dynamic Mode Decomposition (DMD)

Uses a sequence of flow field “snapshot” to construct its projection basis.

The projection basis is comprised of the left singular vectors from a **singular value decomposition** of the snapshot sequence.

These vectors are also known as **Proper Orthogonal Decomposition** modes, or POD modes.

# Modal Decomposition Example



Video: DLR Lampoldshausen.



# Modal Decomposition Example



DMD mode of primary shock oscillation.

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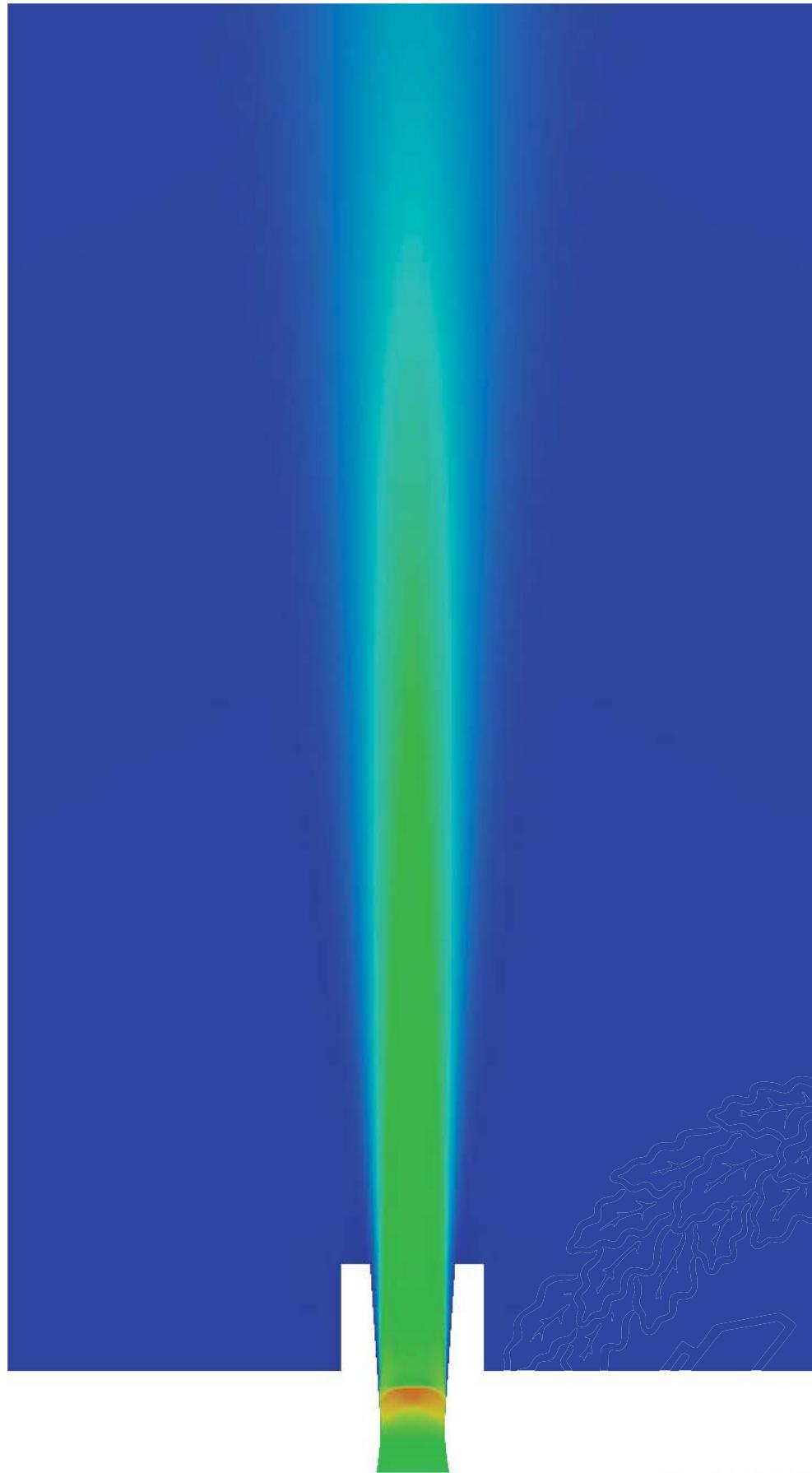


# DMD of axisymmetric URANS Simulations

- DMD was applied to axisymmetric URANS simulations
- Compared well with Arnoldi modes in some cases
- Successfully identified so called *transonic resonance* in a conical nozzle

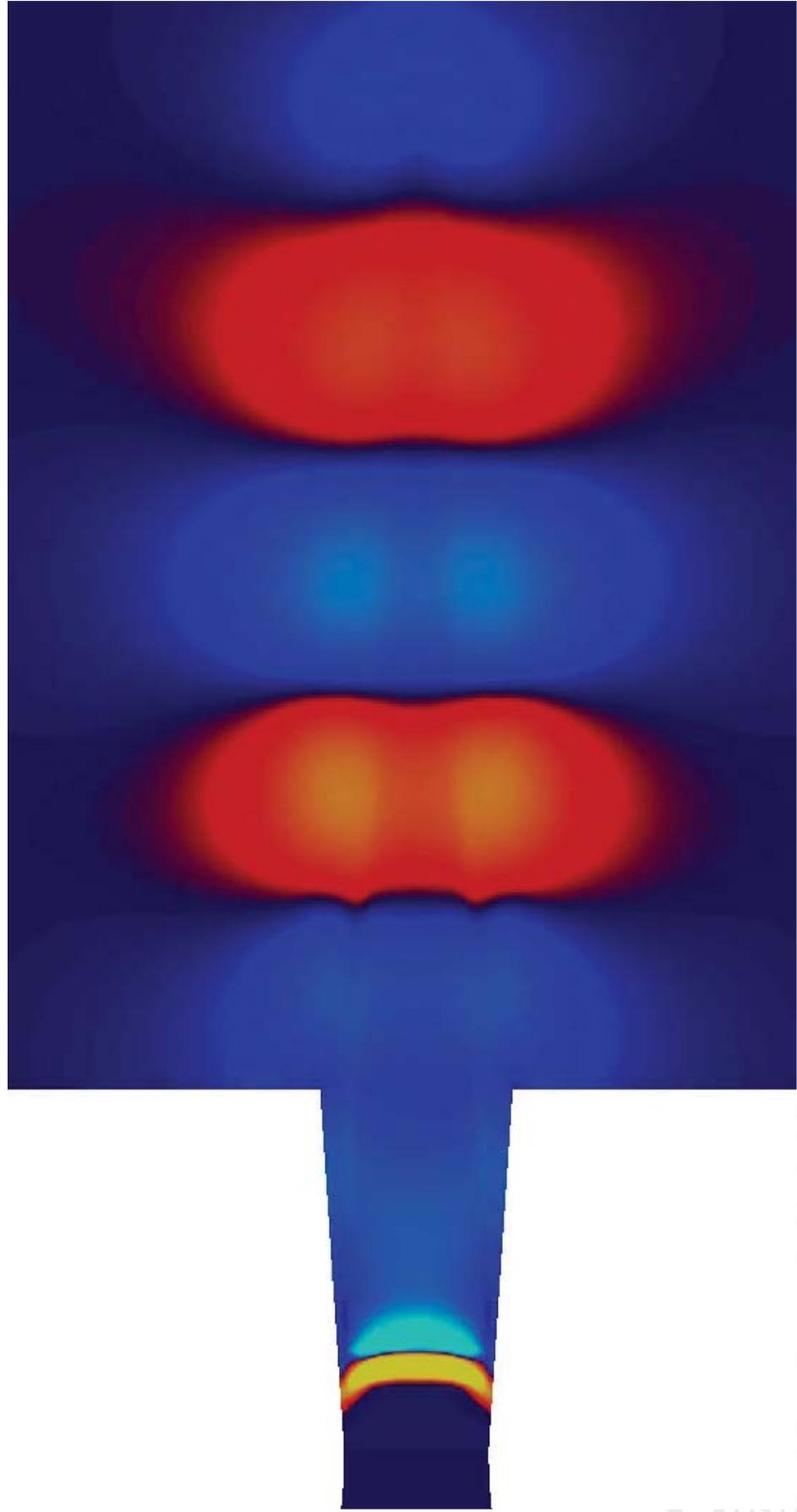
Challenges with noise

# Axisymmetric URANS



Velocity Magnitude.

# DMD - Transonic Resonance



Pressure oscillation field.

# Detached Eddy Simulation (DES)

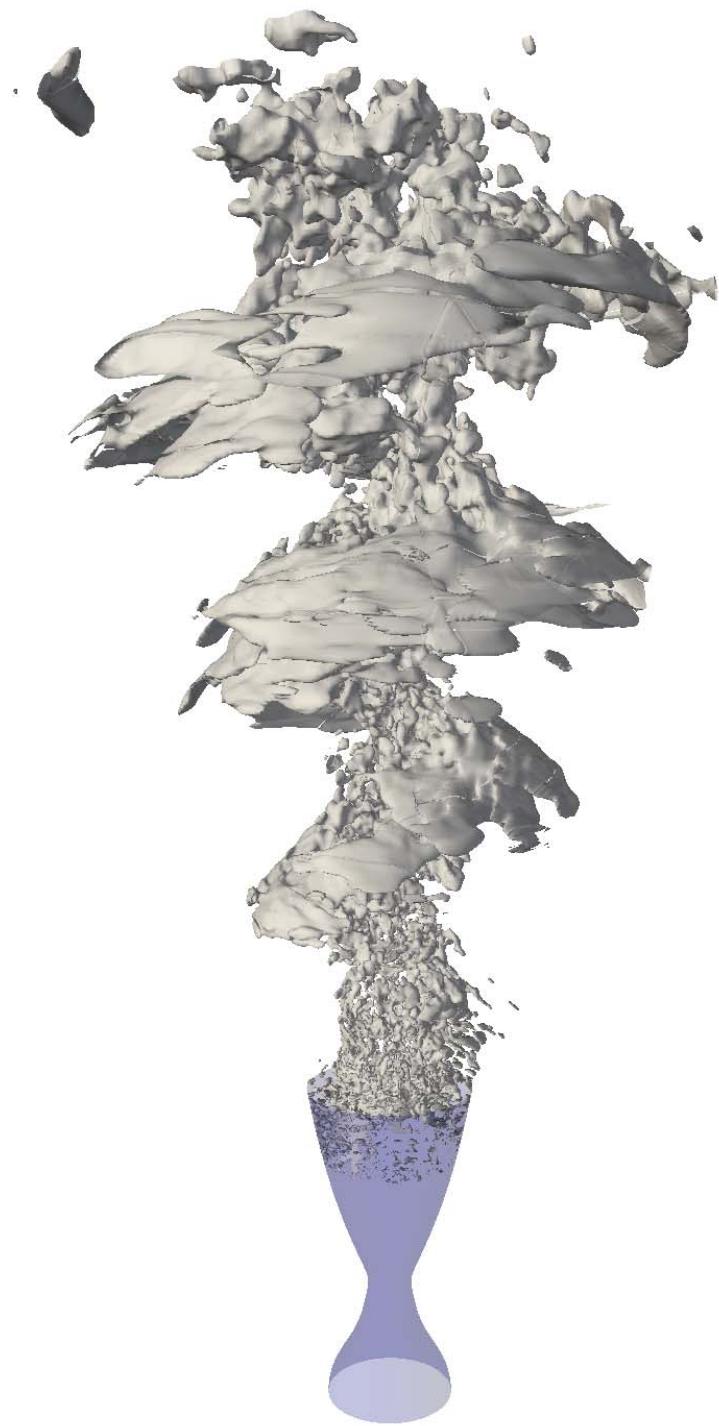
- ▶ DES of a separated nozzle flow
- ▶ Simulated side-loads were lower than measured side-loads but within uncertainty levels
- ▶ DMD Identified modes responsible for **side-loads** and nozzle **ovalization**
  - ▶ Again we had challenges with noise

# Detached Eddy Simulation (DES)



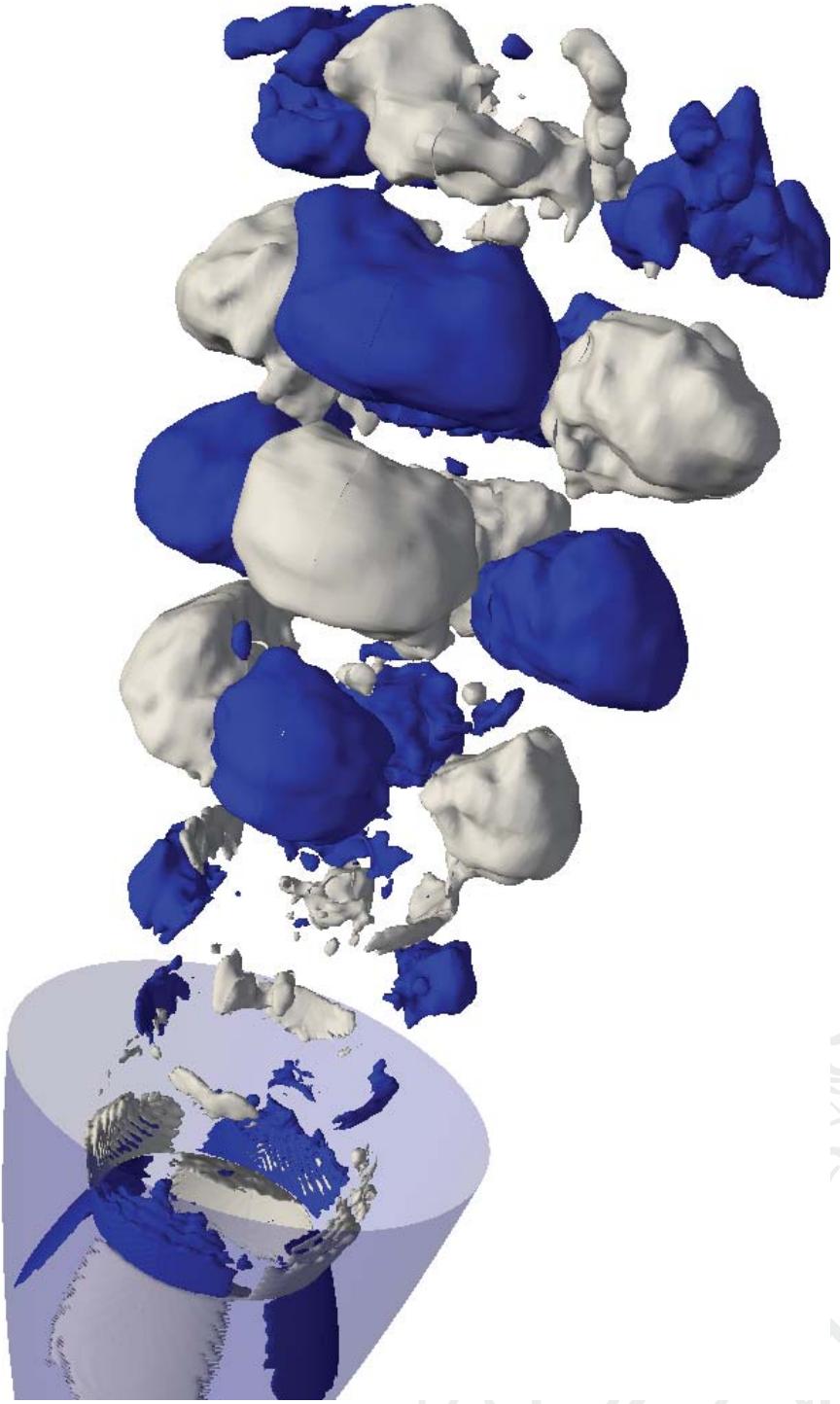
Velocity Magnitude.

# DMD - Helical Mode



Isosurface of pressure oscillation field.

# DMD - Ovalization



Isosurface of pressure oscillation filed.

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# Implicit Compressible Flow Solver

- Semi-implicit Solver
- Implicit Residual Smoothing
- Dual Time Stepping
- Fully Implicit Solver



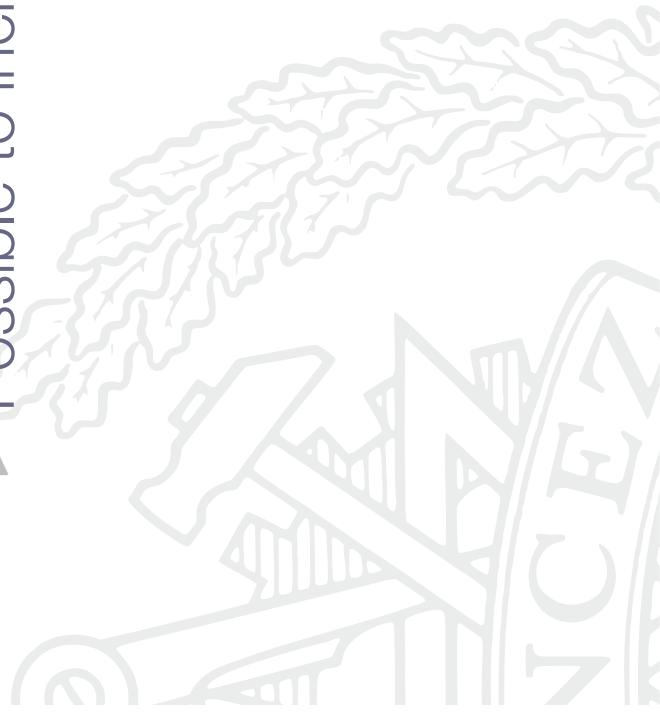
# Semi-implicit Solver

- ▶ Local implicit problem in the wall-normal direction



# Implicit Residual Smoothing

- ▶ Preconditioning
- ▶ Increases the stability margin of the explicit time marching method
- ▶ Possible to increase CFL number



# Dual Time Stepping

- Reformulation of the explicit time stepping algorithm
- Each time step is solved iteratively using sub-iterations
- Dual Time Stepping combined with Implicit Residual Smoothing results in a solver speed-up factor of 10-80 (depending on case)



# Fully Implicit Solver

- ▶ Global implicit formulation
- ▶ Fully coupled problem leads to very large matrices
- ▶ Matrix coefficient calculation
  - ▶ analytic formulations
  - ▶ automatic differentiation
- ▶ Matrix solver based on the solvers and data storage structures in PETSc

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# Summary

- The Arnoldi and the DMD methods can be used to identify important oscillating modes
  - These modes could potentially lock onto structural eigenmodes
- Future challenges are:
- to eliminate the effect from noise in DMD modes
  - to further develop side-load resolving simulation methods

# Publications and Collaborations

- ▶ one PhD student (Lic 2014, PhD 2017)
- ▶ 6 conference papers
- ▶ one journal publication (one additionally under review)
- ▶ exchange with DLR Lampoldshausen (ongoing discussions about future project)
- ▶ joint publication on DMD with INSA Rouen (future exchange)

